Transmission Lines

We have studied wave propagation in unbounded media → media of infinite extent.

Unguided wave propagation → uniform plane wave exists throughout all space and E.M. energy associated → spreads over a wide area.

Wave propagation in unbounded media → radio or TV broadcasting

not useful in situations like telephone conversation.

Another mean of transmitting power or information → Guided structures

↳ Serve to direct the propagation energy from the source to the load.

Examples → Transmission Lines and waveguides

Transmission Lines
- → Power distribution (at low frequencies)
- → Communications (at high frequencies)

Transmission lines → twisted pair and coaxial cables (thinnet & thicknet)

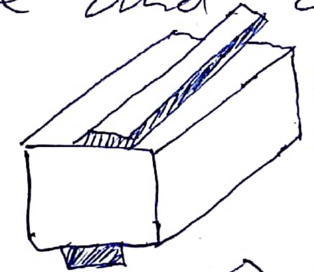
→ used in computer networks → Ethernet and internet

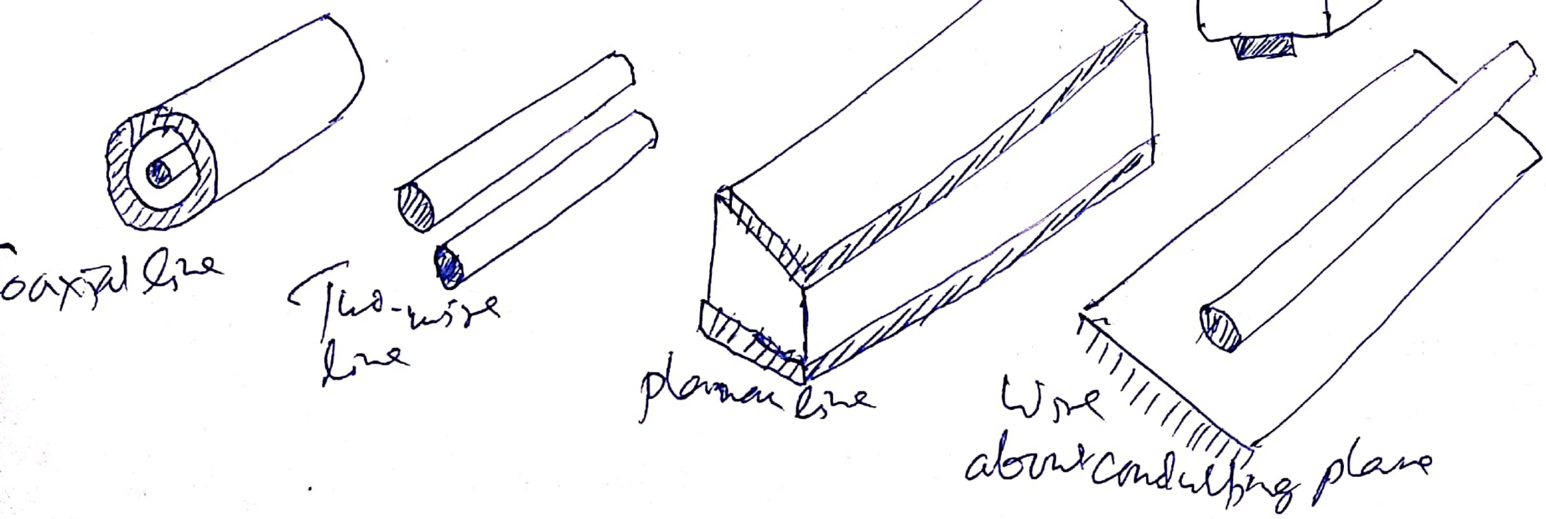
Transmission Line → Consists of two or more parallel conductors used to connect a Source to a load

Ex:

Sources	load
Hydroelectric generator	factory
Transmitter	Antenna
Oscillator	Oscilloscope

Transmission lines include → coaxial cable, a two-wire line, a parallel plate or planar line, a wire about the conducting plane and a microstrip line.





Coaxial cables → used in electrical laboratories

Microstrip lines → Important in integrated circuits
→ metallic strips connecting electronic elements are deposited in dielectric substrates

Transmission line problems → Solved using EM field theory and electric circuit theory.

Transmission Line Parameters

↳ resistance per unit length R
Inductance per unit length L
Conductance per unit length G
Capacitance per unit length C

Each of lines → has specific formulas for finding R, L, G and C

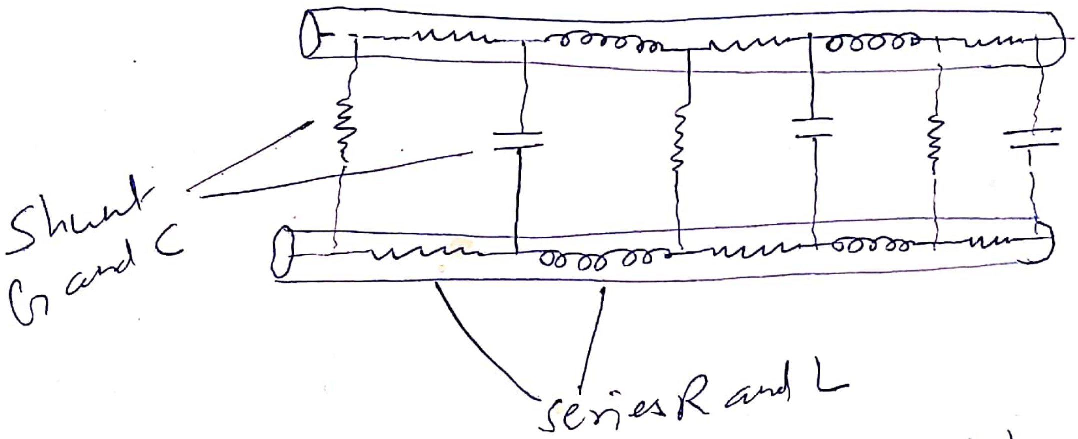
For example for coaxial cable or line

$$R\,(\Omega/m) = \frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right] \qquad L\,(H/m) = \frac{\mu}{2\pi}\ln\frac{b}{a}$$

$$G\,(S/m) = \frac{2\pi\sigma}{\ln b/a} \qquad C\,(F/m) = \frac{2\pi\varepsilon}{\ln b/a}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$$ = Skin depth of the conductor

1. The line parameters R, L, G and C are not discrete or lumped, parameters are uniformly distributed along the entire length of the line



2. The conductors are characterized by σ_c, μ_c, $\varepsilon_c = \varepsilon_0$ and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.

3. $G \neq \frac{1}{R}$

$R \rightarrow$ ac resistance per unit length comprising the line

$G \rightarrow$ conductance per unit length due to the dielectric medium separating the conductors.

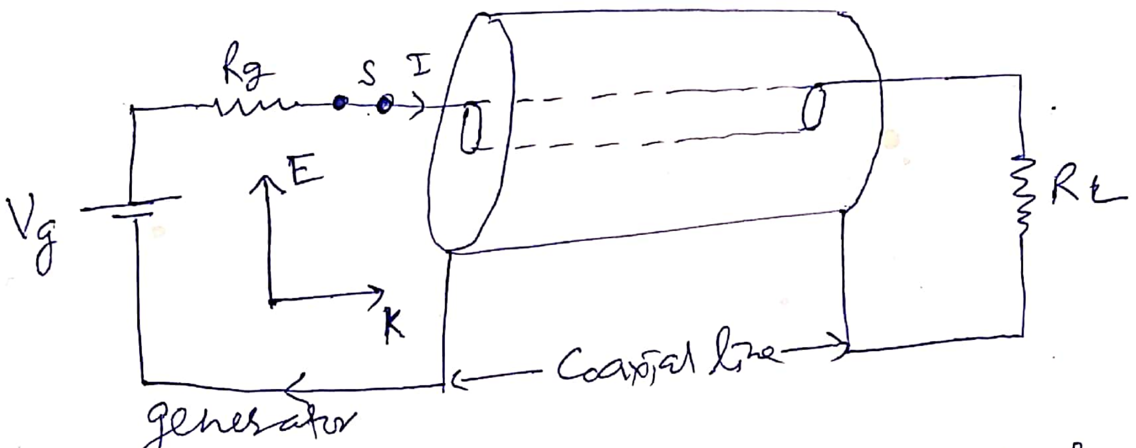
4. $L = L_{ext}$ external inductance per unit length

$L_{in} = \left(\frac{R}{\omega}\right) \rightarrow$ negligible at high frequencies

5. for each line $LC = \mu\varepsilon$ and $\frac{G}{C} = \frac{\sigma}{\varepsilon}$ —(1)

Let us consider how an EM wave ~~propagate~~ propagates through a two-conductor transmission line

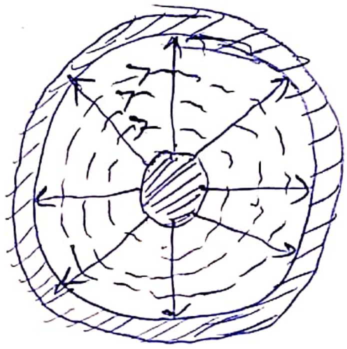
Consider the coaxial line connecting the generator or source to the load



When switch S is closed the inner conductor is made positive w.r.t the outer one so that the field E is radially outward

According to Ampere's law

→ H field encircles the current carrying conductors



Poynting vector (E×H) points along the transmission line.

⇒ Closing the switch → creates a disturbance, which appears as a transverse electromagnetic (TEM) wave propagating along the line.

Transmission Line Equations

A two conductor transmission line supports a TEM wave i.e. electric and magnetic fields on the line are transverse to the direction of wave propagation.

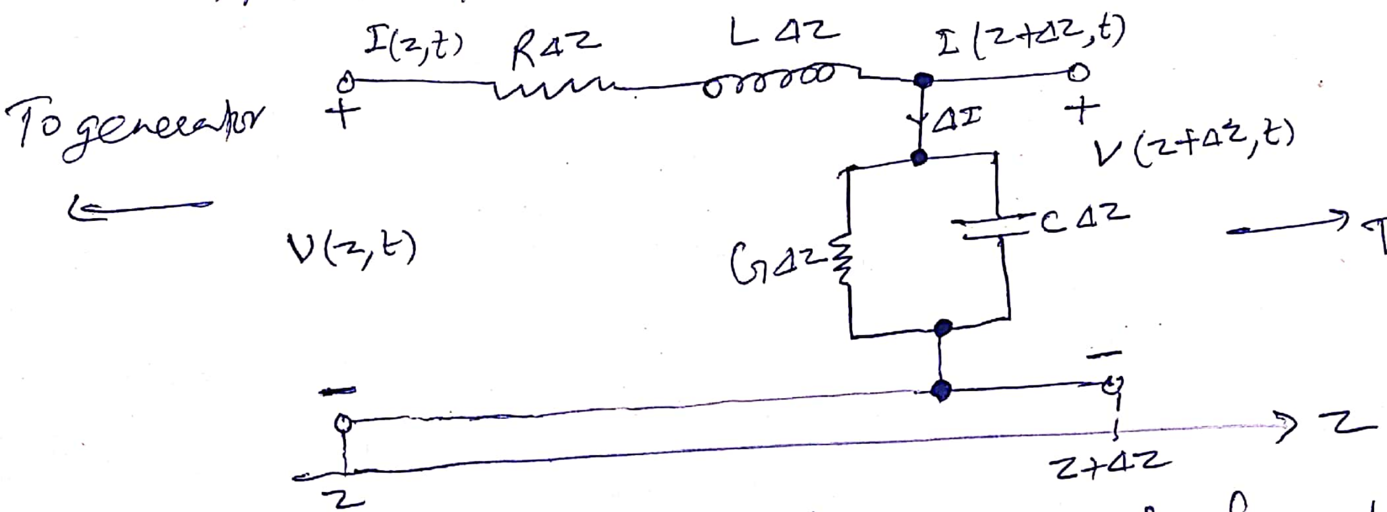
for a TEM wave → fields E and H are uniquely related to V and I respectively

$$V = -\int E \cdot dl \quad , \quad I = \oint H \cdot dl \quad \text{---(2)}$$

We use circuit quantities V and I in solving the transmission line problem

We examine an incremental portion of length Δz of a two-conductor transmission line

We take an equivalent circuit, called the L-type equivalent circuit



Wave propagates along the +z direction from the generator to the load.

By applying Kirchhoff's voltage law to the outer loop of the circuit

$$V(z,t) = R\Delta z\, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

or $$\frac{V(z+\Delta z, t) - V(z,t)}{\Delta z} = R\, I(z,t) + L\frac{\partial I(z,t)}{\partial t} \quad \text{---(3)}$$

as $\Delta z \to 0$

$$\frac{\partial V(z,t)}{\partial t} = R\, I(z,t) + L\frac{\partial I(z,t)}{\partial t} \quad \text{---(4)}$$

Similary, applying Kirchhoff's law to the main node of the circuit

$$I(z,t) = I(z+\Delta z, t) + \Delta I$$
$$= I(z+\Delta z, t) + G\Delta z\, V(z+\Delta z, t) + C\Delta z \frac{\partial V(z+\Delta z, t)}{\partial t}$$

or $$\frac{I(z+\Delta z, t) - I(z,t)}{\Delta z} = G\, V(z+\Delta z, t) + C\frac{\partial V(z+\Delta z, t)}{\partial z} \quad \text{---(5)}$$

as $\Delta z \to 0$

$$\frac{\partial I(z,t)}{\partial z} = G\, V(z,t) + C\frac{\partial V(z,t)}{\partial t} \quad \text{---(6)}$$

If we assume harmonic time dependence so that

$$V(z,t) = \mathrm{Re}\left[V_s(z)\, e^{j\omega t}\right] \quad \text{---7(a)}$$

$$I(z,t) = \mathrm{Re}\left[I_s(z)\, e^{j\omega t}\right] \quad \text{---7(b)}$$

where $V_s(z)$ and $I_s(z)$ $\longrightarrow$ phasor forms of $V(z,t)$ and $I(z,t)$ respectively